## MOTION DIAGRAMS

When first applying kinematics (motion) principles, there is a tendency to use the wrong kinematics quantity - to inappropriately interchange quantities such as position, velocity, and acceleration. Constructing a motion diagram should reduce this confusion and should provide a better intuitive understanding of the meaning of these quantities.

The kinematics quantities can be related as:
Position - the location of an object.
Average velocity - the change in position divided by the change in time (sometimes called the rate of change of position of the object), and
Average acceleration - the change in velocity divided by the change in time (sometimes called the rate of change of velocity of the object).

A motion diagram represents the position, velocity, and acceleration of an object at several different times. The times are usually separated into equal time intervals. At each position, the object's velocity and acceleration are represented by arrows. If the acceleration is constant throughout the motion, one arrow can represent the acceleration at all positions shown on the diagram. We will be drawing motion diagrams of constant acceleration or regions of constant acceleration.

## Procedure:

To draw a motion diagram, you sketch 3 to 5 consecutive "frames" of the position of the object. You start by drawing the actual object at some initial position. Then, you draw a second object at its next position (next frame). For example, if the object is moving to the right, then the next position is to the right. How far it is placed to the right depends on how fast the object is moving - greater separation of the two objects if the object is moving fast, smaller separation of the two objects if the object is moving slow. You should follow this same approach for the next 2 to 4 frames.

To draw the velocity "arrows", look at the separation between two consecutive objects (this is the rate of change of position) and draw the length of the velocity arrow (called the velocity magnitude or speed) based on the position separation. The direction of the velocity arrow is in the direction that the object is moving. We locate the velocity arrow usually below the drawn object at the same location below the drawn object.

To draw the acceleration "arrow" (only one arrow for constant acceleration motion diagrams), look at the separation between the velocity arrows (this is the rate of change of velocity). If the velocity arrows are not changing length and direction, then the acceleration is zero. If the velocity arrows are getting longer, then the acceleration is some constant in the same direction as the velocity arrows. If the velocity arrows are getting shorter, then the acceleration is some constant in the opposite direction to the velocity arrows.

The motion diagrams for three common types of linear motion are described below.
Constant Velocity: The first motion diagram, shown in Fig. 1, is for an object moving at a constant speed toward the right. The motion diagram might represent the changing position of a car moving at constant speed along a straight highway. Each object (car) indicates the position of the object at a different time. The objects are separated by equal time intervals. Because the object moves at a constant speed, the displacements from one object to the next are of equal length. The velocity of the object at each position is represented by an arrow with the symbol $\mathbf{v}$ under it. The velocity arrows are of equal length (the velocity is constant) and in the same direction. The acceleration is zero because the velocity does not change.


Fig 1. The motion diagram for an object moving with a constant velocity. The acceleration is zero because the velocity is not changing.

Constant Acceleration in the Direction of Motion: The motion diagram in Fig. 2 represents an object that undergoes constant acceleration toward the right in the same direction as the initial velocity. This occurs when your car accelerates to pass another car or when a race car accelerates (speeds up) while traveling along the track. Once again, the objects (cars) represent schematically the positions of the object at times separated by equal time intervals $\Delta t$. Because the object accelerates toward the right, its velocity arrows increase in length toward the right as time passes. The product $\boldsymbol{a}(\Delta t)=\Delta \boldsymbol{v}$ represents the increase in length (the increase in speed) of the velocity arrow in each time interval $\Delta t$. The displacement between adjacent positions increases as the object moves right because the object moves faster as it travels right.


Fig 2. A motion diagram for an object that is accelerating in the direction of its velocity. The velocity increases as time progresses.

The direction of the acceleration arrow is in the same direction as the velocity arrows since the speed is increasing.

Constant Acceleration Opposite the Direction of Motion: The motion diagram in Fig. 3 represents an object that undergoes constant acceleration opposite the direction of the initial velocity (this is sometimes called deceleration - a slowing of the motion). For this case, the acceleration arrow points left, opposite the direction of motion. This type of motion occurs when a car skids to a stop. The objects (cars) represent schematically the positions of the object at equal time intervals. Because the acceleration points left opposite the motion, the object's velocity arrows decrease by the same amount from one position to the next. We are now subtracting $\Delta v=a(\Delta t)$ from the velocity during each time interval $\Delta t$.. Because the object moves slower as it travels right, the displacement between adjacent positions decreases as the


Fig 3. A motion diagram for an object whose acceleration points opposite the velocity. The magnitude of the velocity decreases as time progresses.
object moves right. This implies that the acceleration is in the opposite direction to the direction of the velocity.

You should become so familiar with these motion diagrams that you can read a linear-motion problem and draw a reasonable diagram that represents the motion described in the problem.

> Motion Diagram - 1a

| (a) Construct a motion |
| :--- | :--- |
| diagram of a car traveling |
| toward the right at a |
| constant speed. |


| (a) Construct a motion <br> diagram of a car traveling <br> toward the left at constant <br> speed. |  |
| :--- | :--- |
|  |  |
|  |  |

> Motion Diagram - 1c

| (a) Construct a motion <br> diagram of a cart being <br> pulled by a rope to the right <br> at a constant speed. |  |
| :--- | :--- |
|  |  |
|  |  |


| (a) Construct a motion <br> diagram of the air glider at <br> rest. |  |
| :--- | :--- |
|  |  |

> Motion Diagram - 2a

| (a) Construct a motion diagram <br> for a bottle rocket whose burning <br> fuel causes it to move vertically <br> upward at increasing speed. | (b) Construct a motion diagram <br> for the bottle rocket in (a) after its <br> fuel is burned and while it still <br> moves upward but now at <br> decreasing speed. | (c) Construct a motion diagram <br> for a bottle rocket after it has <br> reached maximum height and is <br> now falling down at an increasing <br> speed. |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


| (a) Construct a motion diagram <br> for a ball being thrown straight up <br> while in the person's hand. <br> Assume that the ball starts from <br> rest. | (b) Construct a motion diagram <br> for the ball in (a) after it has left <br> the person's hand and before it <br> reaches maximum height. | (c) Construct a motion diagram <br> for a ball after it has reached <br> maximum height and is now <br> falling down. |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Motion Diagram - 2c


| (a) Construct a motion diagram <br> for a parachute while it is <br> opening. | (b) Construct a motion diagram <br> for a parachute descending at a <br> constant rate. | (c) Construct a motion diagram <br> for a parachute during the process <br> of hitting the ground and coming <br> to a complete stop. |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

> Motion Diagram - 3a

| (a) Construct a motion <br> diagram for a skier moving <br> at a decreasing speed up an <br> incline. |
| :--- | :--- |

> Motion Diagram - 3b

| (a) Construct a motion <br> diagram for a bicyclist <br> coasting up an incline. |  |
| :--- | :--- |
|  |  |

$$
\text { Motion Diagram - } 4
$$

| (a) Construct a motion <br> diagram for a person <br> standing on the floor. |  |
| :--- | :--- |
|  |  |
|  |  |

